

# Application of Queuing Theory to Vehicular Traffic on Nakuru Total Road Stretch

Fredrick Ochieng Odhiambo<sup>a\*</sup>, George Otieno Orwa<sup>b</sup>, Romanus Otieno Odhiambo<sup>c</sup>

<sup>a,c,b</sup>*Department of Statistics and Actuarial Science Jomo Kenyatta University of Agriculture and Technology P. o.*

*Box 62000-, Nairobi 00100, Kenya*

<sup>a</sup>*Email: od032003@yahoo.com*

<sup>b</sup>*Email: gorwa@jkuat.ke.ac*

<sup>c</sup>*Email: romanusodhiambo@jkuat.ke.ac*

## Abstract

Nations strive to avoid losing revenue and human lives through long traffic snarl ups and frequent accidents on the roads. For this reason Considerations must be made to increase the number of lanes or even better to change from a single carriage to more robust dual carriages. However number of lanes and dual carriage alone serve no purpose for the accidents frequencies and traffic snarl ups that appear to defy even the most modern and sophisticated highway designs. Service time for traffic using such roads would need to be improved. Clearly therefore a numerical model is necessary for the road designers and developers to help understand road improvement demands. In this paper we establish the queue model for the Nakuru – Salgaa road Stretch and test the model with real data from the Case Study. Data is collected between the Soil- junction and the Total junction. We derive the arrival rate, service rate, utilization rate and the probability of Bulking using the M/M/1 queuing model. It is estimated that the arrival rate at the Soil- junction is 37 vehicles per minute and at total junction the service rate is 44 Vehicles per minute this does not march the dwindle service rates in section that are now black spots. The average number of vehicles on single road stretch is on average 15 per minute with some sections recording a high of 40 vehicles per minute and the utilization of the sections of stretch is on average 0.8. The benefit of performing the queue analysis for the road stretch is finally discussed and recommendations provided.

**Keywords:** Road Section; Queue; Single carriage; dual carriage; waiting times.

---

\* Corresponding author.

## 1. Introduction

The last 15 years has seen Kenya Government invest very heavily on the road infrastructural improvement. At the time of this study, the government is finalizing the first face of its standard gauge railway to supplement the road infrastructure. Both National and County arterial highways are being developed or rehabilitated. Meanwhile, the traffic volume has increased so rapidly, thanks to the Second hand vehicles imports that have made it easy for vehicle affordability. Unfortunately, road safety situation has been getting worse and road accident rate has been progressively rising with the construction of highways. Some sections of the newly constructed roads are posing new safety challenges. Road safety issue is a complex engineering problem; it is affected by various factors, such as human factor, vehicle performance, road and environment etc. Countries are now paying great attentions to road safety issues. In order to reduce road accident rate, many research work are being carried out, both technical measures and road safety policy have been applied. For improving road safety in the late 80s, UK developed a systematic road safety audit procedure. According to this concept, all safety issues should are examined during the life cycle of a road, including planning, design, construction, maintenance and operation stages . Kenya has since introduced the National Transport and Safety Authority (NTSA). Road queue system identification for purpose of flow improvement is an important technical measure of improving road safety. The main objective of analyzing the queue system is to help establish the queue equilibrium and server utilization at any section  $S_i$  of the road system.

## 2. Literature Review

Efficient transportation system plays an important role in catering for the daily necessities in the lives of the citizens [1]. Movement through the road system is not done for the sake of it but to help people access amenities and services that are central to their lives. Many travels about to attend to social obligation, like going to work, school respond to demanding duties and leisure. According to [2], transportation remains a crucial factor for urban insertion giving access to economic activity, facilitating family life and helping in spinning social networks. Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of such an existing transport system [3].

Consequently, in an attempt to acquire vehicles to enable individuals travel faster and carry out daily activities easily, cities in the world now witness tremendous increase in vehicle population, all against the slow pace of infrastructural expansion. It can be said that the high volume of vehicles, the inadequate infrastructure and the irrational distribution of the development are the main reasons for increasing traffic jam, [4] and perhaps the heart braking road carnages now witnessed in Kenya.

A typical deficiency of existing roads is a lack of conformity of the function of the road in the network with high traffic volumes and the existing usage with mixed traffic [5,8,6]. Considered a scheduling in networks with interference constraints and reconfiguration delays, which may be incurred when one service schedule is dropped and a distinct service schedule is adopted [7] Examined a system consisting of N parallel queues, served by one server under an assumption that time is slotted, and the server is capable of serving one packet per slot.

For road system, customers compete for the time slot. Reference [16] Investigated how many telephone circuits were necessary to provide phone services that would prevent customers from waiting too long for an available circuit. The same can be replicated for the road system. Reference [9] defined the queue building blocks derived basic queuing system. [10] Explained patients flow data through the lens of queuing scientist. Reference [11] derived a data based description of hospitals flow, supplemented with description of various factors that cause delays in hospitals.

A similar approach is adopted for this study. Analysis of accident causes and characteristics of black spots indicate that highway features and traffic devices are certainly the factors affecting road safety [12].

### **2.1 Geometric growth of Kenyan Roads**

While the rate of growth of roads in Kenya is exhibit a geometric growth the of vehicle population tend to exhibit an exponential growth. Road segments or a road system be considered of poor, good, very good or even excellent standards.

When these elements are provided for a road system, traffic flow and road utility is highly improved. Even so, with every other factor in place, there is another significant factor that must be considered, for a busy road that has excelled in other areas. The other most important factor is the queuing time.

The average waiting time and the average number of vehicles that would ordinarily wait for a server in a server system are important measures for a manager [13]. Indeed whereas the management can do little to control the number of vehicle arrival at any given entry point, the road management bear greater on average time vehicles would spend on a server given that a well improved road reduces motorists struggle to go through it.

This study illuminates road design (servers) and management as the first step in increasing service rates in road queues and hence improving queue discipline. Reference [14] Used queuing model to obtain utilization period of a busy restaurant.

### **2.2 Little's theorem**

According to [15] "queuing system" consists of discrete objects we shall call "items" that "arrive" at some rate to the "system." Within the system the items may form one or more queues and eventually receive "service" and exit. According to little law, under steady state condition (equilibrium) the average number of items in a queue system equals the average rate at which items arrive multiplied by waiting time. Little's theorem can be useful in quantifying the maximum achievable operational improvements and also to estimate the performance change, when the system is modified [14].

## **3. Material and methods**

This section introduces the data sources, discusses the M/M/1/ queuing model which this article uses to model

the vehicular traffic flow and to explore how vehicular traffic service could be maximized using queuing theory in order to minimize the rate of road accidents along Nakuru- Salgaa-Total junction of Nakuru.

### Map of Nakuru-Total Junction Road Stretch



**Figure 1:** Map of Nakuru-Total road stretch

### 3.1 Nature of the road system

#### 3.1.1 The road system

This study found out through field observation that waiting lines are a common phenomenon on the Kenya road system. Traffic waits for hours on certain section of Kenya roads. In this respect the Kenyan road A104 is considered. The Section with greater number of black spots, along the A104 road stretch is microscopically examined for their contribution to road carnages. The Salgaa- Total road stretch (segment drawn in red) on the map Figure1, is considered as a case study. This is because of its notorious Cases of road crashes and numerous black spots.



**Figure 2:** Black spots dot the road sections

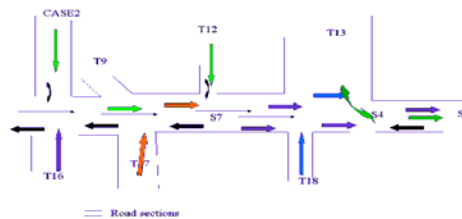
The road section receives traffic from Nairobi, the Capital city and the larger Mt Kenya region destined for both

Uganda and the expansively populated Western and Nyanza. It simultaneously distribute a large volume of both light and heavy commercial traffic to North rift Kenya, Southern Sudan and Uganda through Malaba border point, figure 2. This Scenario without any doubt has rendered the road section, a deadly section both for the local and international drivers.



**Figure 3:** Heavy commercial traffic on the stretch

The road is entirely a single carriage with single lanes in most cases making overtaking a very risky venture for motorist, figure 3.



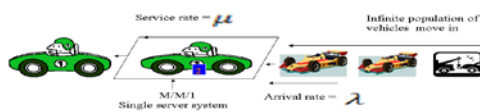
**Figure 4:** Schematic illustration of Nakuru-Total road stretch

The road network studied is composed of nodes and links Figure 4. A node in this case separate parts of the same road with different characteristics for example, a dual carriage way may narrow to a single stream or even several intersections flowing into a carriage way an a point and exits at another point. The road system and traffic schematic diagram generally depicts a process where customers (road users) arrive at a road sections according to some distribution with a mean arrival rate  $\lambda$ . Many sections of the road system are largely single lanes, clearly suggesting a single server system. Customers arrive at sections from a population size that cannot be determined in advance, thus an infinite population. A part from a few cases like the, most modern Thika Superhighway and very few sections of the roads, customers (traffic) arrive in a single lane traffic moving towards a specific direction. Such traffic form single queues.

The number of lanes (servers) is for greater length, single lanes. The capacity of the system is finite. The population,  $P$  from where the system draws clients is infinite, i.e.  $P = \infty$ . The consequences of single lanes for systems are traffic jams and numerous traffic accidents that sometimes results to indefinite traffic delays, figure 5.

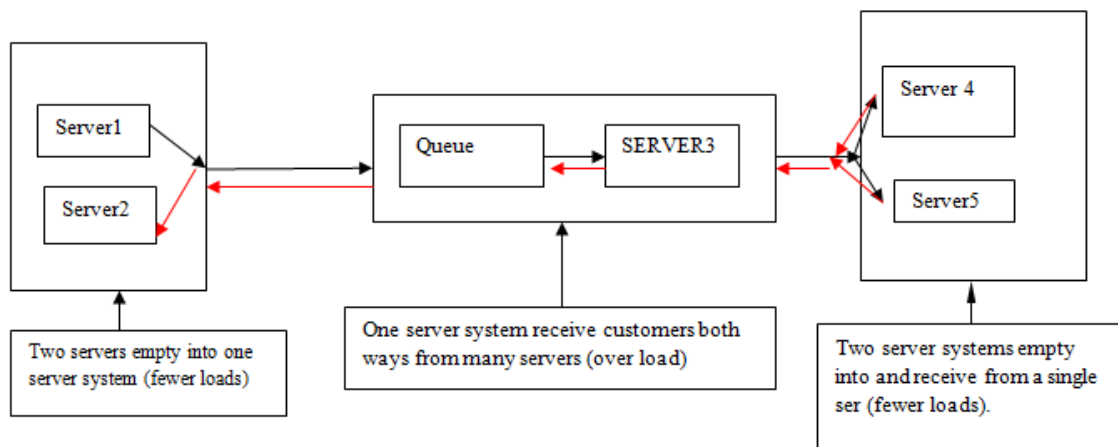


**Figure 5:** Common accidents that cause indefinite queues a long Nakuru-Total road stretch.



**Figure 5:** Single server system with over load

**3.1.2 Queue disciplines illustrated**



**Figure 6:** Single server system with overload

On many lanes, characterizing greater lengths of road sections, the queue discipline is first come first served with, occasional priorities being given to special vehicles on emergency operations and Ambulances, Therefore the Markov queue system M/M/1 with FIFO as the queue discipline is adopted. With this scenario, every traffic try to be ahead of others to, “beat the jam”. Service time for these customers is therefore independent. In figure 6, it is evident that the middle single server receives from a two server system and empty into a two server system. There is no doubt that we have strain and stress (overload) in the middle server.

### 3.1.3 M/M/1 queue model

M/M/1 queuing model means that the arrival and service time are exponentially distributed (Poisson process). For the analysis of road M/M/1 queuing model, the following variables are identified and investigated:

1. Arrival process: Poisson process with rate  $\lambda$ .
2. Service rate is exponential with parameter  $\mu$ .
3. Service time distribution (S) and inter arrival time distribution (A) are independent.
4. The number of servers(s) is 1.
5. The capacity of the system(c) is finite.
6. Size of the population from which the vehicles arrive is infinite.

The state transitions are due to arrival into or departure of customers from the queue. The departures from the queue are as result of successful service or customers leaving the queue. Nearest neighbor transitions are considered here. The state of the process at time,  $t$ ;  $N(t)$ , ( $t \geq 0$ ) a continuous time Markov chain with

With transition rate  $q_{i \rightarrow i+1} = \lambda$ ,  $q_{i \rightarrow i-1} = \mu$ . A Clear situation of a birth death process.

Let  $\pi_i = P[N(t) = i]$

With  $\lim_{t \rightarrow \infty} N(t)$ , with  $\pi_i \geq 0$  and  $\sum \pi_i = 1$

$$\pi_{i-1} = \frac{\lambda}{\mu} \pi_i \quad (1)$$

Taking  $\rho = \frac{\lambda}{\mu}$  and  $\pi_0 = (1 - \rho)p^n$ ,

$$p < 1, n = 1, 2, \dots$$

$P$  is the server utilization, thus the probability the system is not empty (system is busy), sometimes called occupancy. If arrival is Poisson with parameter  $\lambda$  then at any time  $t$  the probability of getting a number  $n$  customers arriving in the system in steady state equilibrium is

$$p_n = \frac{(\lambda t)^n}{n!} e^{-\lambda t}. \quad (2)$$

Where:

- $t$  is used to define the interval 0 to  $t$
- $n$  is the total number of arrivals in the interval 0 to  $t$ .
- $\lambda$  is the total average arrival rate in arrivals/sec.

The probability that there is no vehicle arriving in the system is given by case  $n = 0$ . Such probability is

$$p_o = e^{-\lambda t}. \tag{3}$$

$\rho$  is the offered load or traffic intensity. It is defined as the average arrival rate ( $\lambda$ ) divided by the average service rate ( $\mu$ ). For a stable system the average service rate should always be higher than the average arrival rate. (Otherwise the queues would rapidly race towards infinity) a scenario that characterize a failing system. Thus  $\rho$  should always be less than one. The instantaneous arrival rate may exceed the service rate. However, Over a longer time period, the service rate should always exceed an arrival rate.

Assuming, Poisson arrival process on the highway, stretch as per conditions in section ii(c). If any one of the three conditions is not met, one cannot assume Poisson arrivals, [16].

### 3.1.4 Steady state (Equilibrium)

#### 3.14.1 Average number of customers in the system at time

The average number of vehicles  $N(t)$  in the system (road stretch) in steady state equilibrium is given by  $N = \sum n \rho^n (1 - \rho)$

$$\begin{aligned} &= \rho(1 - \rho) \sum_{n=0}^t n \rho^{n-1} \\ &= \rho(1 - \rho) \frac{1}{(1 - \rho)^2} \\ &= \frac{\rho}{(1 - \rho)} \end{aligned} \tag{6}$$

As  $\rho$  approaches 1 the number of customers would become very large. Clearly  $\rho$  will approach 1 when the average arrival rate starts approaching the average service rate. In this situation, the server would always be busy hence leading to a queue build up (large N).

But Taking,

$$\rho = \frac{\lambda}{\mu} \tag{4}$$

We can write that  $E[N] = \frac{\lambda}{\mu - \lambda}$ , clearly in the event of multiple servers with increased rate of service ( $\mu$ ). The value of N reduces markedly with constant arrival rate ( $\lambda$ ).

#### 3.15.2 Mean number of customers in queue waiting for service

$$E[N] = \frac{\rho^2}{(1 - \rho)} \tag{5}$$

#### 3.15.3 Total (mean) waiting time (system delay)



Let us denote the Total mean waiting time by T. This waiting time shall include time spent in service. We refer to this as the Total system delay.

$$T = \frac{N}{\lambda} = \frac{N}{\lambda} \times \frac{\lambda}{(\mu-\lambda)}$$

$$= \frac{N}{(\mu-\lambda)}, \tag{7}$$

**Table 1:** Traffic Data for Nakuru-Salgaa Stretch

LOCATION	SESSION	ARRIVAL		SERVICE	
		AV. NO. OF CARS	Time(min)	AV. NO. OF CARS	Time(min)
SOILO-SALGAA	MORNING	515	13	672	15
SALGAA-MOLO	MORNING	680	60	700	60
MOLO-TOTAL	MORNING	307	24	400	30
SOILO-SALGAA	AFTERNOON	700	60	757	60
SALGAA-MOLO	AFTERNOON	733	60	800	60
MOLO-TOTAL	AFTERNOON	200	20	360	20
SOILO-SALGAA	EVENING	247	24	300	24
SALGAA-MOLO	EVENING	300	20	430	20
MOLO-TOTAL	EVENING	400	15	410	15
MEAN FOR SECTIONS		453.5	32.8	536.5	33

Waiting time T would thus reduce significantly with increase in  $\mu$ . Since nothing can be done about the rate of average arrival into the system ( $\lambda$ ). Road design can be manipulated to provide for multiple servers in this way increase the service rate consequently reducing the total mean waiting time.

**3.15.4 Data Sources**

This paper uses data recorded within the working periods of morning, afternoon and evening along Nakuru-Salgaa-Total junction of Nakuru, Kenya. The traffic counts were taken at 3 specific stations during three different times of the day; in the morning, afternoon. The data on vehicle arrivals and service time is summarized in table 1.

**3.2 M/M/1/ Queuing System**

This paper therefore focuses on the M/M/1 queuing system. M/M/1 refers to negative exponential arrivals and service times with a single server. The queuing systems under study, much depicts M/M/1 this in its greater length. M/M/1 is a good approximation for a large number of queuing systems. Suitability of M/M/1 queuing is that it is easy to identify from the server standpoint [17].

#### 4. Exponential Arrivals and Service Process

M/M/1 queuing systems assume a Poisson arrival process. This assumption is a very good approximation for arrival process in real systems that meet the following rules:

1. The number of customers in the system is very large.
2. The impact of a single customer for the performance of the system is reasonably large, that is, a single customer influences to a large extent the use of the system resources.
3. All customers are independent, i.e. their decisions to use the system are independent of other users.

The mean vehicular parameters are determined for road section with the following results

Equilibrium It is necessary that such a queue be in equilibrium at any section  $S_i$  for the section to maintain a stationary regime [5]. It is considered that an accident on a given road stretch inversely relates to the level of server utilization. In the event of accident on a road stretch (server) S, the rate  $\mu$  would be greatly reduced as many motorists would enter the stretch unknowingly and without any guarantee of proceeding out. Clearly rate accident  $A_i$ , within a time interval  $[0 \text{ to } t]$  will affect the server (stretch) rate of service  $\mu$ .

**Table 2:** Results for traffic Parameters for other Sections

	Session	Arrivals	Time(s)	$\lambda$	Service	Time(s)	$\mu$	$-\lambda+\mu$	$\rho$
SOILO-SALGAA	M	515	13	39.6	672	15	44.8	5.18	0.88
SALGAA-MOLO	<b>M</b>	<b>680</b>	<b>60</b>	<b>11.33</b>	<b>700</b>	<b>60</b>	<b>11.6</b>	<b>0.33</b>	<b>0.97</b>
MOLO-TOTAL	M	307	24	12.79	400	30	13.3	0.54	0.95
SOILO-SALGAA	A	700	60	11.66	757	60	12.6	0.95	0.9
SALGAA-MOLO	A	733	60	12.2	800	60	13.3	1.11	0.91
MOLO-TOTAL	A	200	20	10	360	20	18	8	0.55
SOILO-SALGAA	E	247	24	10.	300	24	12.5	2.2	0.82
SALGAA-MOLO	E	300	20	15	430	20	21.5	6.5	0.69
MOLO-TOTAL	E	400	15	26.6	410	15	27.33	.66	.957

Using the observation of Soil- Salgaa, the following estimates are obtained.

#### 4. Results

##### 4.1 Morning Session (Soil conservation –Salгаа stretch)

###### A. Calculation

Arrival Rate=  $515/13 = 39$  cars per minute

Service Rate =  $672/15 = 44.8$  cars per minute

Traffic Intensity =  $39.6/44.8 = 0.88$

Mean time spent in the system =  $1 / (44.8(1-0.88))$

= 0.19min

Mean time spent waiting in queue =  $\frac{0.88}{(44.8(1-0.88))} = 1.47min$

Table 2: Results for traffic Parameters for other sections

Mean number of Cars in the system =  $\frac{0.88}{1-0.88} = 7.6$

Mean number waiting in queue =  $\frac{0.88^2}{1-0.88} = 7$

The utilization of the road stretch is at 0.88. When the service rate is higher, the utilization will be lower, which makes the probability of zero traffic in the road stretch is very small as can be derived using (2).

$$P_0 = 1 - 0.88$$

$$= 0.12$$

The generic formula that can be used to calculate the

Probability of having  $n$  customers in the restaurant is as follows,

$$P_n = (1 - 0.88)0.88^n$$

$$= 0.12(0.88)^n$$

The utilization is directly proportional to the mean number of vehicles that use the section of the road at different time of day. It means that the mean number of customers will increase as the utilization increases. The utilization rate at the road stretch is very high at 0.97. This, the of utilization in the morning and can be explained by large traffic volume made up of commuters into town for work or transit vehicles that having

packed in town would now leave early for their destinations. When the service rate is higher the utilization will be lower, which makes the Probability of having many customers waiting in the queue decrease.

**Table 3:** Excel sheet generated results for traffic parameters

	$(\rho)^2$	$1-(\rho)$	Av. cars in system	Av. cars waiting	Av. Queue time	$T_s$	$T_q$	Av. Server time
SOILO-SALGAA	0.781	0.11	7.6	6.75	1.47	0.19	0.17	0.19
SALGAA-MOLO	0.9	0.02	34	33.02	102	3	2.91	3
MOLO-TOTAL	0.92	0.04	23.61	22.6	43.59	1.84	1.77	1.8
SOILO-SALGAA	0.85	0.07	12.28	11.35	12.9	1.05	0.97	1.05
SALGAA-MOLO	0.8	0.08	10.94	10.02	9.79	0.89	0.82	0.89
MOLO-TOTAL	0.30	0.44	1.25	0.69	0.15	0.12	0.069	0.12
SOILO-SALGAA	0.67	0.17	4.66	3.83	2.11	0.45	0.37	0.45
SALGAA-MOLO	0.48	0.30	2.30	1.61	0.35	0.15	0.10	0.15
MOLO-TOTAL	.951	.024	40	39.02	60	1.5	1.46	1.5
Mean for road sections	0.78	0.14	15	14.3	25.8	1.02	0.98	1.02

**Table 4:** Average for all sections

	Arrivals	Time	$\lambda$	Service	Time	$\mu$	$-\lambda+\mu$	Traffic intensity( $\rho$ )
Average for sections	453.5	32.8	16.6	536.5	33	19	2.83	0.85

Every car entering the stretch spends on average 2 minutes per section.

### 5. Conclusion

This research paper has discussed the application of queuing theory of Vehicular Traffic on Nakuru- Total road Stretch. Here we have focused on two particularly common decision variables as a way for introducing and illustrating all the concepts. From the result we have obtained we can conclude that the rate at which vehicles arrive in the queuing system is approximately 16.6 vehicles per minute and the service rate is 19 customers per minute. The traffic intensity index is at a maximum of 0.97 and at a minimum of 0.55 with a general mean of 0.85. This traffic intensity is high and not sustainable on a Single lane two way traffic that characterize the Nakuru- Salgaa road stretch, perhaps this partly explains the frequency of road carnages on the road stretch. It

can be concluded that the arrival rate will be greater the morning hours and the in the late afternoon service rate will be greater if more lanes or most preferably a dual carriage is introduced to address the traffic intensity. The Kenya government should urgently consider making the Nakuru-Salgaa-Total junction a dual carriage in this way, waiting time and queue conflicts will be reduced.

## **6. Recommendation**

- There is need for future research to investigate the contribution of the background of the road user in terms their familiarity with the road terrain and its contribution to the frequent accident on the stretch.
- This research can help The Kenya National Highway Authority, National Traffic Safety Authority (NTSA), and The Traffic Police increase safety on Kenyan roads.
- The result of this paper work may become the reference to analyze the current road system and improve it by making structural adjustment or overhauling the current road system and replacing it with a multi-lane one way traffic.
- The finding based on the server utility by place and Time is useful to commuters to plan their activities and avoid being held in the traffic snarl ups.
- The formulas that were used during the completion of the research are applicable for future research.

Road and Transport authorities to work closely with the police to secure vital data that go hand in hand with accident occurrence. The relevant, up to date and meaningful database is lacking. Statistics of road accidents that point out other road elements associated with the crashes is lacking. The traffic enforcement agencies cannot provide detailed data both related to road and road traffic dynamics. Their inclination is more on legal aspect. Lack of cooperation from the traffic police was disturbing. Very crucial data could not be obtained due their classification as “sensitive”. Despite the assistance from NACOSTI, the study required much more in terms of equipment. Such were expensive and hard to come by.

## **Acknowledgements**

We wish to express our sincere gratitude to the team of enumerators that helped with data collection. Thanks to the County Governments for cooperating with us to grant permit to access the county offices for data and National council for Science and Technology (NACOSTI) for the financial support and Permit to carry out this Study.

## **References**

- [1] Intikhab, QA., Huapu, L., & Shi Y. “Urban Transportation and Equity: A Case Study of Beijing and Karachi”. *Transportation Research Part A: Policy and Practice*, 42: 1, (2008).
- [2] Wane, H.R. “Urban Mobility and Emissions: Towards Accurate Standards for Sub Saharan Africa.” A Research Program and Results on a Sahelian Case: The District of Bamako. At the Session on Air pollution, Accra, Ghana, 18-20, (2008).

- [3] Banks J., Carson J. S., Nelson B. L., Nicol D. M., "Discrete-Event System Simulation", Prentice Hall International Series. 3<sup>rd</sup> edition (2001)
- [4] Odero W, Khayesi M and Heda PM." Road traffic injuries in Kenya: magnitude, cause and status of intervention". *Journal of Injury Control and Safety Promotion*, 10:53–61.(2003).
- [5] Antov D. "Road Safety Inspection Guidelines and Checklists" , Tallinn University of Technology" Final version: .31 (2012)
- [6] Dharmawirya M. and Ali E. "Case study for restaurant queuing model." *International Conference on Management and Artificial Intelligence IPEDR* vol.6 (2011)
- [7] Krishna Jaggannathan and Eytan Modiano, "The Impact of Queue Length Information on Buffer Overflow in Parallel Queues," *IEEE transactions on Information Theory*, (2013).
- [8] A. K. Kharwat, "Computer Simulation: an Important Tool in the Fast-Food Industry," *Proceedings of the 1991 Winter Simulation Conference*, IEEE Press, pp. 811-815.(1991)
- [9] Luke, BC. And Richard, S. "The economic costs of road traffic crashes: Australia, states and territories." *Journal of Accident analysis and prevention.*., Volume: 38, Issue: 6, Pages: 1087-1093 Pub Med: 16797462 (2006).
- [10] Armony M, Israelit S, Mandelbaum A.: "On Patients flow in Hospitals": A data based queue science Perspective, Rambam Hospital, ORT Braude college& Mayo clinic, IBM research (2015)
- [11] VipinJain, Ashlesh Sharma, and Lakshminarayanan Subramanian. "Road Traffic Congestion in the Developing countries", (2012).
- [12] Jayaram, KS and Lincoln, W.; "The relative impact of consignee behavior and road traffic congestion on distribution costs." *Journal of Transportation Research Part B: Methodological*, Elsevier Ltd. Volume: 41, Issue: 9, Pages: 10.1016. (2007).
- [13] Guo Z., Gao J., Kong L. ; "The road safety situation investigation and characteristics analysis of black spots of arterials highways", *Advances in Transportation Studies an international Journal Section A 1* (2003).
- [14] Lefebvre M., "Applied stochastic processes" by Springer publishers (2000),
- [15] Little, J. D. C. "A Proof of the Queuing formula:  $L = AW$ ," *journal of Operations Research*, 9, (3)383-387. (1961)
- [16] Event Helix, "M/M/1 queuing system"., from Erlang A.K, *The theory of possibilities and telephone conversation*" (1909), Retrieved January 16, 2013

- [17] Anokye M., “Application of Queuing Theory to Vehicular Traffic at Signalized Intersection in Kumasi-Ashanti Region, Ghana.” *American International Journal of Contemporary Research* Vol. 3 No. 2013, 2:2-3 (2013)